

Stage di ANPOBASSO - 2012

Titolo nota

23/02/2012

Esercizi

1) Per quali m interi $m^2 + 15m - 12$ è multiplo di 13?

2) In quanti modi 8 persone possono sedersi attorno a un tavolo (ROTONDO) sapendo che due hanno litigato e non vogliono stare vicini?
(idem con una fila di posti al cinema)

$$1) \quad m^2 + 15m - 12 \equiv 0 \pmod{13}$$

$$m^2 + 2m + 1 \equiv 0 \pmod{13}$$

$$(m+1)^2 \equiv 0 \pmod{13}$$

$$13 \text{ divide } (m+1)^2 \Leftrightarrow 13 \text{ divide } m+1$$

$$m+1 \equiv 0 \pmod{13} \quad m \equiv 12 \pmod{13}$$

$$m = 13k + 12 \\ k \text{ intero}$$

$$2) \quad 7! - 2 \cdot 6! = 5 \cdot 6! \quad \text{in ordine}$$

$$8! - 2 \cdot 7! = 6 \cdot 7! \quad \text{in file.}$$

ARITMETICA

$$1102 \leq m \leq 2011 \quad \text{T.c.} \quad (m+1)^2 - m^2 = p^2 \quad p \text{ primo.}$$
$$2m+1 = p^2$$

$$2205 \leq 2m+1 \leq 4023$$

$$47 \leq p \leq 63 \quad 47, 53, 59, 61$$

$$\frac{47^2-1}{2} + \frac{53^2-1}{2} + \frac{59^2-1}{2} + \frac{61^2-1}{2}$$

$$24 \cdot 46 + 27 \cdot 52 + \underbrace{30 \cdot 58 + 30 \cdot 62}_{30 \cdot 120} =$$
$$= \quad \quad \quad =$$

p, x, q di cui 2 sono primi, 1 è un quadrato.

$$t^2$$

$$\frac{t^2-1}{\quad}, t^2, \frac{t^2+1}{\quad}$$

PRIMI

$$t^2 - 1 = (t-1)(t+1)$$

∴

$$t-1=1 \Rightarrow t=2$$

$$\Rightarrow \boxed{3, 4, 5}$$

1, 2, 3

2, 3, 4

60

———— ✱ ————

4, 10, 11, 14, 16, 23, 25, 32, 34

1 1 2 2 1 2 1 2 1
✱ ✱ ✱ ✱ ✱ ✱ ✱

↓ ↓ ↓ ↓ ↓
1 2 1 2 1 2 1 2 1
↑↑ ↑↑ ↑↑ ↑↑

5! · 4! = 120 · 24 = 12² · 2 · 10 = 2880

———— ✱ ————

1! + 2! + 3! + 4! + ... + 2011! ≡ ? mod 576

576 = 2⁶ · 3²

8! = 2 · 4 · 6 · 8 · 3 · 5 · 7
2⁷ · 3 · 3 · 5 · 7 = 2⁷ · 3² · ...

1 + 2 + 3 · 2 + 24 + 120 + 720 + 5040
—————
—————
153

———— ✱ ————

n f.c. 10ⁿ - 1 ≡ 0 mod 11 → (-1)ⁿ ≡ +1

20ⁿ + 1 ≡ 0 mod 11 → (-1)ⁿ ≡ -1

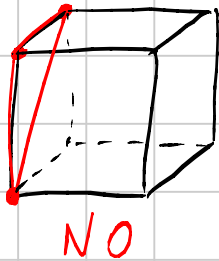
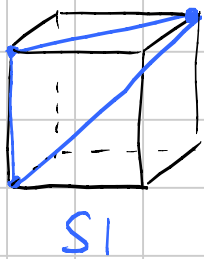
$$10 \equiv -1 \pmod{11}$$

$$\frac{\# \text{ pari tra } 1 \text{ e } 2011}{\# \text{ dispari tra } 1 \text{ e } 2011} =$$

$$= \frac{1005}{1006} = 0,9990 \dots$$

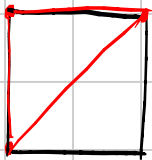
COMBINATORIA

①



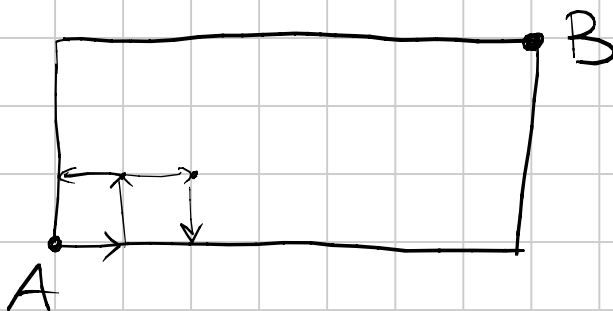
8 vertici totali
3 " triangolo

$$\binom{8}{3} - 6 \binom{4}{3} = 32$$



$$\binom{4}{3}$$

②

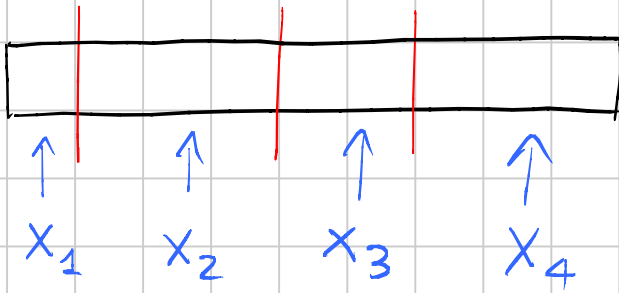


$$\begin{matrix} 7 & \rightarrow \\ 3 & \uparrow \end{matrix} \binom{7+3}{3} = \binom{7+3}{7} = \binom{10}{3}$$

③

$$X_1 + X_2 + X_3 + \dots + X_k = n \quad \begin{matrix} \text{a) } X_i \geq 1 \\ \text{b) } X_i \geq 0 \end{matrix}$$

a



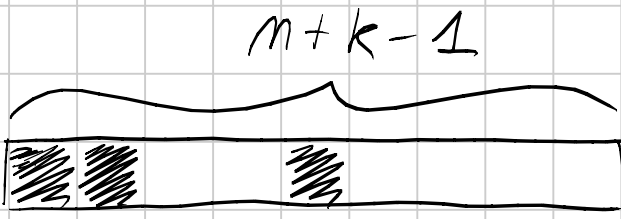
$k = 4$

$m = 9$

$\forall i$

$$\binom{m-1}{k-1}$$

b



$m \neq 0$

$$x_1 + x_2 + \dots + x_k = m$$

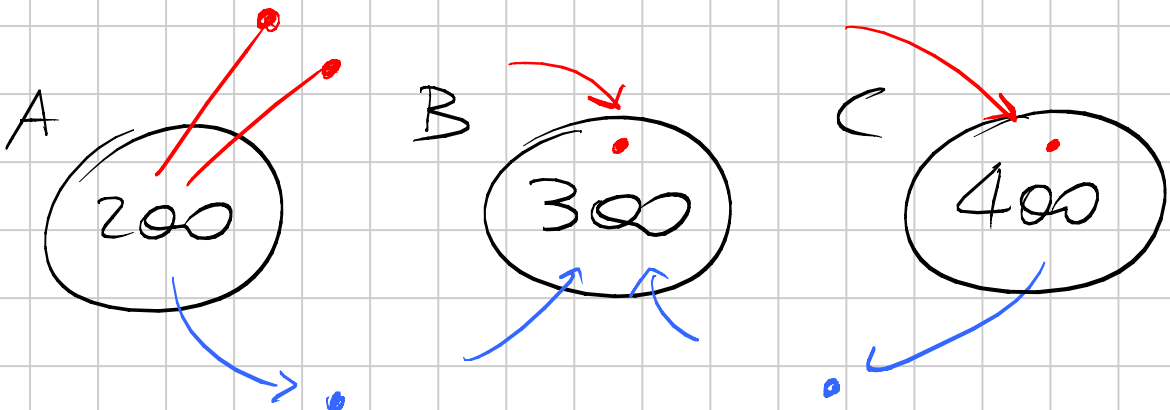
$k-1 \quad 0$

$k = 4$


$m = 9$

$$\binom{m+k-1}{k-1}$$

4



NO

$B - A : 100$ 
diminuisce di 3
rimane invariata
aumenta di 3

da 100 non riesco ad arrivare 0